

Section 6.2

Cardinality

If A is a finite set, then its cardinality is $n(A)$ = the number of elements in A .

Cardinality of a Union

If A and B are finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

In particular, if A and B are disjoint ($A \cap B = \emptyset$) then $n(A \cup B) = n(A) + n(B)$.

Cardinality of a Complement

If S is a finite universal set and A is a subset of S , then

$$n(A') = n(S) - n(A).$$

Cardinality of a Cartesian Product

If A and B are finite sets, then

$$n(A \times B) = n(A)n(B).$$

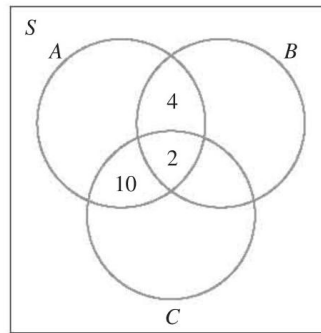
Problem 1. Find the cardinality of the following sets:

- a) $A = \{n | n \text{ is a positive even integer less than } 10\}$
- b) S if S is the set of outcomes when three indistinguishable coins are tossed.
- c) B if B is the set of outcomes when three distinguishable coins are tossed.

Problem 2. Let $A = \{Dirk, Johan, Frans, Sarie\}$, $B = \{Frans, Sarie, Tina, Klaas, Henrika\}$, $C = \{Hans, Frans\}$. Find the following:

- a) $n(A \cup B)$
- b) $n(A \cup (B \cap C))$

Problem 3. Use the given information to complete the partially solved Venn diagram.



$$\begin{aligned}n(A) &= 20, n(B) = 20, \\n(C) &= 28, n(B \cap C) = 8, \\n(S) &= 50\end{aligned}$$

Problem 4. Your favorite restaurant offers a total of 14 desserts, of which 8 have ice cream as a main ingredient and 9 have fruit as a main ingredient. Assuming that all of them have either ice cream or fruit or both as a main ingredient, how many have both?

Problem 5. On a particularly boring transatlantic flight, one of the authors amused himself by counting the heads of the people in the seats in front of him. He noticed that all 37 of them either had black hair or had a row to themselves (or both). Of this total, 33 had black hair and 6 were fortunate enough to have a whole row of seats to themselves. How many black-haired people had a whole row to themselves?

Problem 6. Records at a small college show the following about the enrollments of 100 first-year students in mathematics, fine arts, and economics: 38 take math, 15 take math and economics, 42 take fine arts, 9 take math and fine arts, 20 take economics, 12 take math and economics but not fine arts, and 4 take economics and fine arts.

- Draw a Venn diagram that represents this information.
- How many take none of these three courses?
- How many students take either math or economics?

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #6, 11, 21, 28, 32, 35, 47, 55